Exponential growth is so closely associated with microbiology that it is easy to imagine being waist deep in *E. coli* after only a few days of global warming. Yet it can be a challenge to understand what exponential growth actually means. In the “How Do You Know a Microbe When You See One” activity in Chapter 1, the growth of bacteria in a test tube is indicated by an increase in the turbidity of the media in the tube. In this activity we will consider the growth of microbes that aren’t measured by turbidity in a test tube, but which are visible to the naked eye—the lowly mold. Using either computer software or manual methods, you will measure the growth of mold on a single slice of bread over the course of ten days. You will then investigate several standard models of growth and consider what these models indicate about the growth of the mold in your cultures.

**Overview of the data collection procedures**

A full description of the procedures for this lab, including mold plate preparation, image collection, image transfer, image measurement, and image analysis, is available in the “Mold Lab Procedures” section of the Appendix and on the CD.

**Establishing the mold cultures**

Begin by setting up two series of cultures: *Aspergillus niger* and *Penicillium chrysogenum*. Different species of mold may be used; we have used *Aspergillus niger*, *Mucor hiemalis* (+) and (-), *Penicillium chrysogenum*, *Penicillium digitatum*, *Penicillium italicum*, *Rhizopus stolonifer* (+) and (-), and *Sordaria fimicola*. If you do not have the facilities to grow and photograph your own mold, photographic datasets of *A. niger*, *P. chrysogenum*, and *M. hiemalis* (+) are provided on the *Microbes Count!* CD.

**Collecting your data**

Once you have established your mold cultures, you will need to record the growth of the mold. To do this you will photograph the cultures at set intervals over a series of days, beginning immediately after you inoculate the bread with the mold. A partial data set of images from *Mucor hiemalis* is shown in Figure 1.

![Figure 1: A partial data set for the bread mold *Mucor hiemalis* (+). Days 2–10 are shown; the area of mold growth has been been darkened for emphasis.](image)

Next, use the photographs of the mold culture to measure the area of mold growth on the bread slices. You can estimate the area of growth in one of two ways. The preferred way is by quantitative image analysis using a freeware tool such as NIH...
Image (Macintosh) or Scion Image (PC). Alternatively, you can use an overlay of transparent graph paper with a manual counting method. You will need to consider how much repetition and averaging you want to use in your data collection. Instructions for both the image analysis and the manual counting methods are in the Appendix and on the CD.

Models of growth

After you have collected your mold growth data, you can use the Excel worksheet file called “Mold Growth Models” to investigate three different models of growth. This worksheet models growth of a mold colony based on user-entered parameter values and models of linear, exponential, and logistic growth. The program will compare the predictions of these growth models with actual data and plot the results graphically.

Introduction to the models

Growth is a key property of many systems: an economic expansion, the formation of a crystal, a religion’s increasing congregation, an adolescent’s growth spurt, a disease epidemic, and the condensation of a stellar mass. All of these are instances of the same basic process. As a result, we can explore the general properties of these very different events using just three simple mathematical models.

Linear growth

The simplest growth model describes linear growth, in which something grows at a constant rate over time. For example, if each day you put two soda cans in a recycling bin, the number of cans in the bin will grow linearly. Linear growth is described by the equation

\[ N(t + 1) = N(t) + D. \]

where \( N(t) \) represents the numbers or size of the system at time \( t \), \( N(t + 1) \) represents the system’s numbers or size of the system one time unit later, and \( D \) is the system’s (linear) growth rate. See Figure 2a.

Figure 2. Examples of different growth models, comparing the experimental data from *Penicillium chrysogenum* to the theoretical curve for each type of growth. These examples are taken from the Mold Growth Models worksheet.

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Exponential growth

Another simple model describes exponential growth, in which something grows at a constant proportional rate over time. For example, if your savings account carries a 5% interest rate, your balance will grow exponentially. Exponential growth is described by the equation

\[ N(t + 1) = N(t) + r \cdot N(t), \]

where \( r \) is the system’s (exponential) growth rate. Note that both linear and exponential growth imply continual expansion: either would take the system to infinite size if continued indefinitely. In practice, therefore, each of these growth patterns generally lasts only for a limited time. See Figure 2b.

Logistic growth

A third model, describing logistic growth, is similar to exponential growth except that it assumes an intrinsic sustainable maximum, sometimes called the carrying capacity. A system far below its carrying capacity will at first grow almost exponentially; however, this growth gradually slows as the system expands, finally bringing it to a halt precisely at the carrying capacity. More complex behaviors can result if the system’s growth rate is high (May, 1976). A system starting above its carrying capacity, by contrast, will decrease ever more slowly until it again reaches carrying capacity. This behavior is modeled by the equation

\[ N(t + 1) = N(t) + \frac{r \cdot N \cdot (K - N)}{K}, \]

where \( K \) is the system’s carrying capacity. Most biological growth processes (as well as many non-biological ones) grow logistically. See Figure 2c.

Using the “Mold Growth Models” worksheet

Open the “Mold Growth Models” file, located in the Modeling Mold folder on the Microbes Count! CD.

The red-bordered cells under each growth model contain values for the model’s parameters. You can replace these default values with other values and observe the effect on the models plotted below. Note that models 1 and 2 have only two parameters, while model 3 has a third (carrying capacity).

Enter your mold growth data, or the data from an existing experiment, in column B, under the “Actual Data” heading. See Figure 3 on the next page.

The worksheet uses the equations above to calculate the area of the mold colony in square millimeters given the parameter values you have chosen. Under each model, it then calculates the summed squared deviations of that model’s predictions from the actual data; the smaller this value, the better that model fits the data. Finally, each model is graphically plotted against the data, first individually and then in a combined plot. See Figure 2d.
Points to note:

- The sum of squared deviations describes how well each model fits the data under the assumption of the particular parameter values entered. Highly inaccurate parameter values will make even the correct growth model seem a poor fit to the data. Since parameters are user-entered, there is no guarantee that they represent a “best fit” for that model. So be sure you can justify the parameter values that you have chosen.

- Model 3 (logistic growth) is essentially a special case of model 2 (exponential growth) with one extra parameter (carrying capacity). Adding parameters to any model will always improve that model’s fit to the data unless the fit is already perfect: it’s therefore important to determine if the improvement in fit justifies the more complicated model. Standard statistical procedures such as the chi-square test can be applied to answer this question, but they lie outside the scope of this exercise.

Using the “Mold Growth Model” worksheet, consider your growth data under three different sets of assumptions:

1. Growth is independent of the population size that already exists; or
2. Growth is proportional to the population size that already exists; or
3. Growth is both
   a) proportional to the population size that already exists and
   b) the ratio of the population size to the growth rate of the bread mold (which decreases as the population increases).

Consider how these assumptions are related to the following questions:

- Is the growth linear? Exponential? Logistic?
- How could you tell?
Conclusion
We often use colloquial language to describe quantitative behaviors. As we observed at the beginning of this activity, many microbiologists refer to the growth of bacterial or viral or fungal populations as exponential growth. However, as you’ve worked with the growth models you have seen that it may be difficult to know if they mean the early phase of logistic growth (pre-leveling off) or if they truly intend an exponential model. We hope that this modeling exercise has helped introduce you to: (a) precision of language, (b) experience in building model complexity by sequentially adding more assumptions or constraints, and, (c) the conversion of verbal, numerical, and pictorial descriptions into symbolic expressions that can be evaluated analytically. We encourage you to repeat these observations or to pool data with classmates in order to improve your ability to accept that the data’s fit to the model you have chosen is not due to random effects. What statistical tests can you use? Can you translate your experience in model building here to other laboratories that you have performed?

Software Used in this Activity

NIH Image
Developed by Wayne Rasband at the U.S. National Institutes of Health and available for download at: http://rsb.info.nih.gov/nih-image/
Platform Compatibility: Macintosh only

Scion Image
Developed by Scion Corporation and available for download at: http://www.scioncorp.com
Platform Compatibility: Windows only

Microsoft Excel®
Platform Compatibility: Macintosh and Windows

Additional Resources
Available on the Microbes Count! CD

Text and Data
A PDF copy of this activity, formatted for printing
The “Mold Growth Models.xls” Excel model file
“Mold Lab Procedures” document
Photographic data sets for three different kinds of mold
Related *Microbes Count!* Activities

Chapter 2: Shaped to Survive

Chapter 3: Modeling Microbial Growth

Chapter 7: Valuing Variegated Variation: Using Natural Experiments to Understand Viral Infection

Chapter 8: Exploring Microbial Fermentation with Korean Kimchee

*Unseen Life on Earth* Telecourse

Coordinates with Video I: The Microbial Universe

Relevant Textbook Keywords

Exponential Growth, Growth, Growth curve, Growth Models, Linear Growth, Logistic Growth, Mold, Mold Spores

Related Web Sites (accessed on 4/20/03)

Dresback, M. K. *Fun with Logs*  
http://www.unf.edu/~tbratina/kylelog.htm

*Microbes Count!* Website  
http://bioquest.org/microbescount

Unseen Life on Earth: A Telecourse  
http://www.microbeworld.org/htm/mam/is_telecourse.htm

References


http://rsb.info.nih.gov/nih-image/about.html

Bibliography

http://math.bu.edu/odes (web site with supplemental materials)


Robie, J. *The Exciting Calculus of Bread Mold* On the web at:  
http://isolatium.uhh.hawaii.edu/m206L/student%20projects/mold.htm

Figure and Table References

Figure 1. Tia Johnson and Joanna Cramer
Figure 2. Anton E. Weisstein
Figure 3. Anton E. Weisstein