Mathematical and Computational Biology Workshop
Sweet Briar College, Sweet Briar Virginia

Biodiversity Module

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Species Diversity

The ***biosphere*** is the layer of life that blankets the Earth. It is composed of an enormous variety of organisms. Tropical rain forests are singled out as the most diverse ecosystems, but just looking out any window you will see a variety of plants and animals. A small patch of grass, on closer inspection, is found to be a mixture of plants with insects and other organisms crawling around. ***Biodiversity*** refers to the variation seen in life at many levels: ecosystem, community, species, and genetic.

Maintaining the diversity of ecosystems is important to the balance of nature as well as to human economics. The pharmaceutical industry is dependent on plants as sources for products. Human food resources are tied to biodiversity through the variety of agricultural plant products. Even tourism has taken a focus on diverse ecosystems as desirable destinations.

Ecosystems are defined as all the living things in an area, their interactions, and the environment in with which they interact. Species diversity is a parameter of ecosystems that measures the variety of organisms. There are some general trends observed in diversity. Diversity increases as one moves from the Polar Regions to the equator. Diversity is higher in more heterogeneous environments than in those that are homogeneous. Diversity may also be related to disturbance. The intermediate disturbance hypothesis says that highest diversity is achieved at intermediate levels of disturbance. For example in prairie ecosystems, highest diversity is maintained with periodic burning.

Several indices have been developed to measure diversity. A simple index is the total number of species in a system. More complex indices take into account not only the ***richness*** of species but also the ***evenness*** of their distribution. Thus a system in which many species are evenly distributed is more diverse than a system in which they are a few abundant species and many rare species. The more complex measures of diversity have their roots in information theory and probability theory.

A Mathematical Formulation of Species Diversity.

Suppose there are M species, which we will label as species 1, 2, …, M. Suppose that there are n_i individuals in species ***i***. Let N denote the total number of individuals, so

$$N = n_1 + n_2 + \cdots + n_M \,.$$

The proportion of individuals in species i is then $p_i = \dfrac{n_i}{N}$.

The **Shannon index of diversity** H (Wilson and Bossert, 1971, p. 144 - 148; Shannon, 1948) is defined by

$$H(p_1, p_2, \ldots, p_M) = -\sum_{i=1}^{M} p_i \ln(p_i) \,.$$

In physics and information theory, H is also known as the entropy function. The function H defies a simple heuristic derivation, but is related to a generalized weighted geometric mean. Some of the mathematical properties of H can be explored with the accompanying Maple worksheet.

Another diversity index is Simpson's index of diversity (Simpson, 1949). Let D denote the probability that two randomly selected individuals from the entire population are from the same species. If the sampling of the two individuals is done with replacement, (or if the population is infinite) then

$$D = \sum_{i=1}^{M} p_i^2 \ ,$$

because the probability of selecting two individuals of the i^{th} species is p_i^2. D is sometimes known as the index of concentration (Simpson, 1949). If the sample is taken from a finite population without replacement, then

$$D = \sum_{i=1}^{M} \frac{n_i(n_i - 1)}{N(N-1)} \ .$$

Since D is a measure of concentration, its complementary probability can be considered to be a measure of dispersion. So, let $S = 1 - D$, which we shall call the **Simpson index of diversity.** Note that S is the probability that two randomly selected individuals from the population **are not** from the same species.

Species Diversity
Basic Activity Using the EXCEL Spreadsheet

1) Examine the relationship between number of species and diversity. You can remove species from the ecosystem by setting the slider to 0.

a) What is the maximum for the Shannon-Wiener index for several values of M (M = # species)?

b) What is the range of values for Simpson's Index for different values M?

2) Examine the relationship between evenness and species diversity. You can vary the relative proportions of species by using the slider bars. Keep the number of species the same.

a) As you experiment at what setting do you get maximum diversity?

b) Are there other settings that give the same value?

c) Do these settings apply to one or the other or both indices (Shannon-Wiener, Simpson)?

Species Diversity Module
Glossary of Mathematical Terms

$\sum$ is the Greek sigma symbol. It is a "sum" sign and used to write sums in a concise form without having to list all the summands.

Example: $\sum_{i=1}^{4} i = 1 + 2 + 3 + 4$.

"*i*" serves as a counter and stops as soon as the number on the top of the sigma symbol is reached.

Note: *i* is not the only summand possible. Usually *i* will appear as an index in the summand instead.

Example: $\sum_{i=1}^{6} n_i = n_1 + n_2 + n_3 + n_4 + n_5 + n_6$

Also, the number of summands does not have to be a specified value. It can be represented by a variable:

Example: $\sum_{i=1}^{N} n_i = n_1 + n_2 + n_3 + ... + n_{N-1} + n_N$

Often, we use a generalized formula if it is not known or not important with which summand we start and how many we include.

Example: $\sum n_i$

Note: The expression following a sum sign is grouped unless parentheses are used to indicate a different order of operations.

Example: $\sum p_i^{\,2}$ means $\sum\left(p_i^{\,2}\right)$, NOT $\left(\sum p_i\right)^2$

Natural Logarithm, $\ln e$:

If you need to solve an exponential equation for the exponent, you use the logarithm. So **the logarithm is an exponent**.

$a^x = c$ is equivalent to $x = \log_a c$

The **natural logarithm** is a logarithm that uses the Euler number ***e* = 2.7182818284…** (pronounce "Oiler number") for its base.
The convention for this logarithm is that $\log_e x$ is written as $\ln x$.

p_i stands for a proportion.

Species Diversity
Glossary of Biological Terms

Biodiversity: variety of life forms on Earth. Biodiversity is evident at various levels, for example: ecosystem, species, and genetic

Biosphere: Layer of living things that envelops the Earth

Ecosystem: Living things and their environment in a given area

Species: population of organisms that interbreed under natural conditions.

Species diversity: measure of variety of species in a given area
Components: Richness = number of species
Evenness = distribution of numbers among the various species

Species Diversity Module
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